

# Supersymmetry breaking as a quantum phase transition

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We explore supersymmetry breaking in the light of a rich fixed-point structure of two-dimensional supersymmetric Wess-Zumino models with one supercharge using the functional renormalization group (RG). We relate the dynamical breaking of supersymmetry to an RG relevant control parameter of the superpotential which is a common relevant direction of all fixed points of the system. Supersymmetry breaking can thus be understood as a quantum phase transition analogous to similar transitions in correlated fermion systems. Supersymmetry gives rise to a new superscaling relation between the critical exponent associated with the control parameter and the anomalous dimension of the field – a scaling relation which is not known in standard spin systems.

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Supersymmetry is an important guiding principle for the construction of models beyond the standard model of particle physics, as it helps parametrizing the problem of a large hierarchy of scales, supports the unification of gauge couplings, and facilitates an intricate combination of internal symmetries with the Poincaré group. On the other hand, supersymmetry must be broken in nature in a specific way without violating these attractive features and other constraints imposed by the standard-model precision data. In addition to a variety of breaking mechanisms often involving further hidden gauge and particle sectors, dynamical supersymmetry breaking [1] can play a decisive role in the formation of the low-energy standard model. Generically, symmetry breaking is related to collective condensation phenomena, often requiring a nonperturbative understanding of the fluctuation-induced dynamics.

From the viewpoint of statistical physics, symmetry breaking and phase transitions between ground states of a differently realized symmetry are often related to fixed points of the renormalization group (RG). In addition, the quantitative properties of the system near a phase transition are determined by the RG flow in the fixed-point regime. As the understanding of symmetry breaking phenomena in quantum field theory and particle physics has often profited from analogies to statistical systems, this paper explores supersymmetry breaking in the light of phase transitions and critical phenomena.

As a simple example, we concentrate on Wess-Zumino models with  $\mathcal{N} = 1$  supercharges in two dimensions in Euclidean spacetime. Dynamical supersymmetry breaking in this and many other supersymmetric systems is governed by a control parameter of the superpotential which plays the role of a bosonic mass term. This is very reminiscent to phase transitions in strongly correlated fermionic systems: here, symmetry breaking is governed by a composite bosonic order parameter, e.g., a superfluid condensate, the mass term of which serves as the control parameter of the phase transition. For instance, the inverse interaction strength of attractively interacting electrons on a honeycomb lattice governs the phase

transition between the semimetallic and the superfluid phase as discussed in the context of ultracold atoms [2] or graphene [3]. As this transition occurs at zero temperature, the critical value of the control parameter marks a quantum critical point of a quantum phase transition.

A standard approach to quantum phase transitions in strongly correlated fermion systems is the Hertz-Millis theory [4, 5] where composite bosonic fields are introduced by a Hubbard-Stratonovich transformation. Subsequently, the fermionic degrees of freedom are integrated out, leaving a purely bosonic description which is dealt with in a local approximation. As this strategy is not always sufficient, a treatment of fundamental fermionic and composite bosonic degrees of freedom on equal footing beyond the Hertz-Millis theory has recently proved successful [6].

Now, supersymmetric systems by definition are combined bosonic-fermionic systems with a high degree of symmetry. A treatment of these degrees of freedom on equal footing is even mandatory to preserve supersymmetry manifestly. Apart from this symmetry, we will show that supersymmetry breaking has many similarities to a quantum phase transition in strongly correlated fermion systems. Beyond these similarities, supersymmetry invokes additional structures which go beyond those known in statistical physics.

Euclidean two-dimensional  $\mathcal{N} = 1$  Wess-Zumino models can be defined by the Lagrange density in the off-shell formulation  $\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}}$  with

$$\mathcal{L}_{\text{kin}} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{i}{4} \bar{\psi} \not{\partial} \psi - \frac{i}{4} \partial_\mu \bar{\psi} \gamma^\mu \psi - \frac{1}{2} F^2, \quad (1)$$

$$\mathcal{L}_{\text{int}} = \frac{1}{2} W''(\phi) \bar{\psi} \gamma_* \psi - W'(\phi) F, \quad (2)$$

with superpotential  $W(\phi)$ , bosonic field  $\phi$ , fermionic fields  $\psi$  and  $\bar{\psi}$  and auxiliary field  $F$ . The prime denotes the derivative with respect to  $\phi$ , and we use a chiral representation for the  $\gamma$  matrices. This model allows for dynamical supersymmetry breaking if the highest power of the bare superpotential is odd.

We use the functional RG to calculate the effective

action  $\Gamma$  of the theory. The functional RG can be formulated as a flow equation for the *effective average action*  $\Gamma_k$  [7]. This is a scale-dependent action functional which interpolates between the classical action  $S = \int(\mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}})$  and the full quantum effective action  $\Gamma$ . The interpolation scale  $k$  denotes an infrared (IR) regulator scale, such that no fluctuations are included for  $k \rightarrow \Lambda$  (with  $\Lambda$  being the UV cutoff), implying  $\Gamma_{k \rightarrow \Lambda} \rightarrow S$ . For  $k \rightarrow 0$ , all fluctuations are taken into account such that  $\Gamma_{k \rightarrow 0} \rightarrow \Gamma$  is the full quantum solution. The effective average action can be determined from the Wetterich equation [7]

$$\partial_t \Gamma_k = \frac{1}{2} \text{STr} \left\{ \left[ \Gamma_k^{(2)} + R_k \right]^{-1} \partial_t R_k \right\}, \quad t = \ln \frac{k}{\Lambda}, \quad (3)$$

which defines an RG flow trajectory in theory space with  $S$  serving as initial condition. Here,  $\left( \Gamma_k^{(2)} \right)_{ab} = \frac{\overrightarrow{\delta}}{\delta \Psi_a} \Gamma_k \frac{\overleftarrow{\delta}}{\delta \Psi_b}$  where the indices  $a, b$  summarize field components, internal and Lorentz indices, as well as space-time or momentum coordinates, i.e.,  $\Psi^T = (\phi, F, \psi, \bar{\psi})$ . The regulator function  $R_k$  is introduced in the form of a quadratic contribution to the classical action,  $\Delta S_k = \int \Psi^T R_k \Psi$ .  $R_k$  can thus be viewed as a momentum-dependent mass term, implementing the IR suppression of modes below  $k$ . Different functional forms of  $R_k$  correspond to different RG schemes. Physical quantities do not depend on the regulator.

For a manifestly supersymmetric flow equation, we derive the regulator from a  $D$  term of a quadratic superfield operator, similarly to our construction for supersymmetric quantum mechanics [8]. The most general supersymmetric cutoff action in the off-shell formulation in momentum space is given by

$$\Delta S_k = \frac{1}{2} \int d^2 p (r_1 \bar{\psi} \gamma_* \psi - 2r_1 \phi F + p^2 r_2 \phi^2 + r_2 \bar{\psi} \not{p} \psi - r_2 F^2).$$

The regulator shape functions  $r_{1,2} = r_{1,2}(p^2)$  determine the precise form of the Wilsonian momentum shell integration. The form of  $\Delta S_k$  guarantees that the whole RG trajectory remains in the hypersurface of supersymmetric action functionals even within approximative calculations; see [9, 10] for further supersymmetric RG studies.

In general, the full quantum effective action consists of all possible operators which are compatible with the symmetries of the theory. In this paper, we use a supercovariant derivative expansion to systematically classify these operators. Truncating the expansion at a given order constitutes a consistent approximation scheme. In a variety of bosonic and fermionic systems, such expansions have been proven to capture the physics of phase transition and critical phenomena quantitatively, see, e.g. [11]. For our quantitative results presented below, we observe a satisfactory convergence: leading-order results generically receive  $\mathcal{O}(1 - 10\%)$  corrections at next-to-leading order, increasing up to  $\sim 50\%$  for observables in the infinite-coupling limit.

Let us start with the lowest-order derivative expansion, corresponding to a local-potential approximation for the superpotential. In this truncation, the effective action is approximated by

$$\Gamma_k = \int d\tau \left[ \mathcal{L}_{\text{kin}} + \frac{1}{2} W_k''(\phi) \bar{\psi} \gamma_* \psi - W_k'(\phi) F \right] \quad (4)$$

with a  $k$ -dependent superpotential  $W_k$ . Projecting the flow equation (3) onto this truncation leads to the flow equation for the superpotential. For a simplified regulator choice with  $r_1 = 0$  and  $r_2 = (|k/p| - 1) \theta(1 - (p^2/k^2))$ , the superpotential flow reduces to

$$\partial_t W_k(\phi) = - \frac{k^2}{4\pi} \frac{W_k''(\phi)}{k^2 + W_k''(\phi)^2}. \quad (5)$$

This regulator choice is convenient for analytical studies. We have verified explicitly by using a variety of different regulators that all conclusions drawn below from this simple choice also hold for other regulators.

The corresponding flow equation for the dimensionless potential  $w_k = W_k/k$  admits a variety of nontrivial fixed-point solutions: already a polynomial expansion for  $\mathbb{Z}_2$  antisymmetric superpotentials to order  $n$ ,

$$w_k' \equiv \frac{\partial w_k}{\partial \phi} = \lambda(\phi^2 - a^2) + b_4 \phi^4 + b_6 \phi^6 + \dots + b_{2n} \phi^{2n}, \quad (6)$$

reveal  $n$  independent solutions to the fixed-point equation  $\partial_t w_k = 0$ , in addition to the Gaussian fixed point [20]. The latter has infinitely many relevant directions in agreement with perturbative power-counting in two dimensions. The number of relevant RG directions decreases for the other fixed points, culminating in a maximally IR-stable fixed point with only one relevant direction. This relevant direction exactly corresponds to the parameter  $a^2$  in the superpotential, and it is common to all other fixed points as well. As  $a^2$  is related to the bosonic mass term, we introduce a critical exponent  $\nu_W$  for this direction, such that  $a^2$  scales as  $a^2 \sim k^{-1/\nu_W}$  at a fixed point. This critical exponent thus corresponds to the negative inverse of the associated eigenvalue of the stability matrix. The exponent  $\nu_W$  plays a similar role for the superpotential, as the exponent  $\nu$  for the effective potential in Ising-type systems. At leading order in the derivative expansion, we obtain from Eq. (5):  $\nu_W|_{\text{LO}} = 1$ . Fixed-point potentials and critical exponents at next-to-leading order will be discussed below.

In order to clarify the role of this relevant direction and its relation to supersymmetry breaking, we study the phase diagram of a Gaussian Wess-Zumino model at infinite volume. This is defined in terms of a quadratic perturbation of the Gaussian fixed point at the UV cutoff  $\Lambda$ :  $W_\Lambda' = \bar{\lambda}_\Lambda(\phi^2 - \bar{a}_\Lambda^2)$ . The dimensionless control parameter  $\delta$  and coupling  $\gamma$  can be associated with

$$\delta := \Lambda^{-1} \bar{\lambda}_\Lambda \bar{a}_\Lambda^2, \quad \gamma := 2\Lambda^{-1} \bar{\lambda}_\Lambda, \quad (7)$$

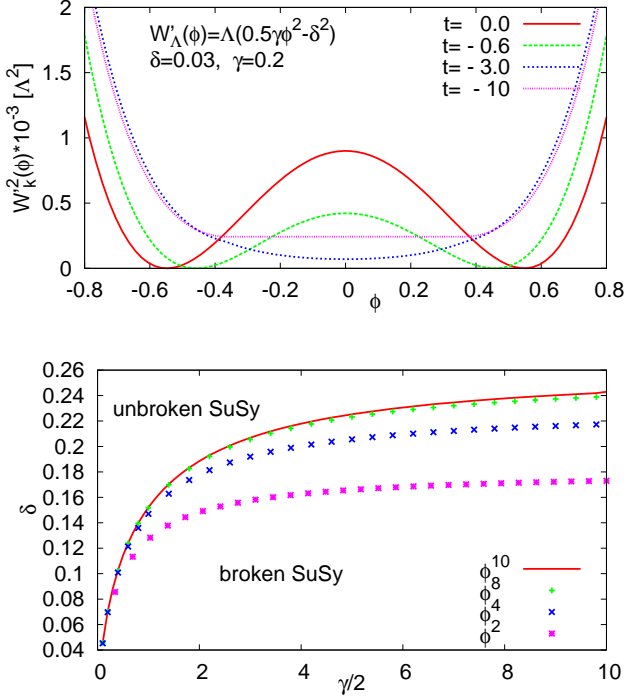


FIG. 1: *Top*: Typical flow of the superpotential in the supersymmetry broken phase. *Bottom*: Phase diagram in the coupling-control-parameter plane for the Gaussian Wess-Zumino model. The lowest set of data points corresponds to the lowest-order result (9). Higher orders converge rapidly.

where the scale is set by the UV cutoff  $\Lambda$ . In the following, we determine the region for the parameters  $\delta, \gamma$  for which supersymmetry is dynamically broken in the IR. A criterion for supersymmetry breaking is provided by a nonvanishing ground state energy, given by the minimal value of  $V(\phi) = \frac{1}{2}(W'_{k \rightarrow 0}(\phi))^2$ . As a true phase transition only occurs in the infinite-volume limit [12], the ground state energy can only be identified at  $k \rightarrow 0$ . Four snapshots of a typical flow of the superpotential in the broken phase are depicted in Fig. 1 (top) for  $\delta = 0.03$  and  $\gamma = 0.2$ .

For a simple estimate of the phase diagram, we truncate the superpotential at order  $\phi^2$  in Eq. (6). From Eq. (5), we obtain the flow of the dimensionless couplings

$$\partial_t a^2 = \frac{1}{2\pi} - \frac{6\lambda^2 a^2}{\pi}, \quad \partial_t \lambda = -\lambda + \frac{6\lambda^3}{\pi}, \quad (8)$$

which can be solved analytically. The phase transition curve in the  $(\gamma, \delta)$  plane is given by the initial values for which  $a^2|_{k \rightarrow 0}$  changes sign. This condition yields

$$\delta = \frac{\arcsin(\alpha)}{\sqrt{24\pi}\alpha}, \quad \alpha^2 = 1 - \frac{\pi}{3\gamma}. \quad (9)$$

Beyond this simple estimate, the phase transition curve can, of course, be identified by a full numerical integration of the flow equation. Already upon inclusion

of higher orders of Eq. (6), we observe a rapid convergence of the phase boundary; see Fig. 1 (bottom). There exists a critical value for  $\delta_{\text{cr}}$  characterizing the phase transition in the strong coupling limit  $\gamma \rightarrow \infty$ . Our best estimate from a numerical higher-order solution is  $\delta_{\text{cr}} \simeq 0.263$ ; (the lowest-order result from Eq. (9) is  $\sqrt{\pi/96} \simeq 0.181$ ). We conclude that supersymmetry can never be broken dynamically above this critical value. This agrees qualitatively with earlier results in the literature [12, 13, 14, 15]. Of course, the numerical value for  $\delta_{\text{cr}}$  is not universal but regulator-scheme dependent, simply because the quadratic initial conditions near the Gaussian fixed point are nonuniversal. A quantitative comparison with other estimates of this critical value, e.g., taken from the lattice, is thus only meaningful if this scheme dependence is accounted for.

The mass of the lowest bosonic excitation is given by the curvature of the full renormalized effective potential at the minimum. In the supersymmetric phase, fermionic and bosonic masses are identical and nonzero. In the broken phase, the lowest fermionic excitation is a Goldstino with vanishing mass. The latter holds also at finite  $k$  in the supersymmetry-broken regime. As the flow in the IR is attracted by the maximally IR-stable fixed point, the bosonic mass in the fixed-point regime is governed by the only relevant direction  $a^2$ . As the minimum of the effective potential in the broken regime is necessarily at a vanishing field, the bosonic mass yields

$$m_k^2 = 2k^2\lambda^2|a^2| \sim k^{2-\frac{1}{\nu_W}}, \quad (10)$$

where the latter proportionality holds in the vicinity of the IR fixed point where  $\lambda \rightarrow \text{const.}$  and  $a^2$  is governed by the critical exponent  $\nu_W$ . For  $\nu_W > 1/2$ , the bosonic mass scales to zero upon attraction by the maximally IR-stable fixed point. We conclude that the broken phase remains massless in both bosonic and fermionic degrees of freedom.

On the other hand, the limit  $k \rightarrow 0$  is an idealized IR limit. Any real experiment as well as any lattice simulation will involve an IR cutoff scale  $k_m$  characterizing the measurement, e.g., the scale of momentum transfer, detector size or lattice volume. Any measurement is therefore not sensitive to  $k \rightarrow 0$  but to  $k \rightarrow k_m > 0$ . We conclude that a measurement of the bosonic mass in the broken phase will give a nonzero answer proportional to the measurement scale, whereas the goldstino will be truly massless. The bosonic mass in both phases as a function of the control parameter  $\delta$  is depicted in Fig. 2 (top) for a coupling of  $\gamma = 0.2$ .

At next-to-leading order in the derivative expansion, the fields acquire a wave function renormalization  $Z_k$ , which is a multiplicative factor of the kinetic term in Eq. (1),  $\mathcal{L}_{\text{kin}} \rightarrow Z_k^2 \mathcal{L}_{\text{kin}}$ . The flow of  $Z_k$  gives rise to an anomalous dimension,  $\eta = -\partial_t \ln Z_k^2$ . Convergence of the derivative expansion requires  $\eta \lesssim \mathcal{O}(1)$ .

# nodes	$\eta$	$\nu_W$
0	0.4386	1.2809
1	0.20	1.11
2	0.12	1.06

TABLE I: Critical exponents of the first fixed points.

From the polynomially expanded superpotential flow at next-to-leading order we can derive the following flow of the renormalized parameter  $a^2$ :

$$\partial_t a_t^2 = \frac{1}{2\pi} \left(1 - \frac{\eta}{4}\right) - \left(1 - \frac{\eta}{2}\right) a_t^2 - \frac{a_t^2}{\lambda_t} \partial_t \lambda_t. \quad (11)$$

At any fixed point with  $\partial_t \lambda \rightarrow 0$ , we can read off that  $a^2$  again denotes a relevant direction with scaling  $a^2 \sim k^{-1/\nu_W}$ , where

$$\nu_W = \frac{2}{2 - \eta}. \quad (12)$$

This remarkable *superscaling* relation connects the superpotential exponent  $\nu_W$  with the anomalous dimension  $\eta$ . Recall that in Ising-like systems the thermodynamic main exponents (i.e.,  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$ ) are related among each other by scaling relations, and can be deduced from the correlation exponents  $\nu$  and  $\eta$  by hyperscaling relations. Beyond that, there is no general relation between  $\nu$  and  $\eta$ . The superscaling relation (12) thus represents a special feature of the present supersymmetric model and is an exact relation to next-to-leading order in the supercovariant derivative expansion. Beyond next-to-leading order, Eq. (12) might receive corrections, since higher-derivative operators involving the auxiliary field can still take influence on the flow of the superpotential. Nevertheless, even if Eq. (12) receives higher-order corrections, the most important result that a potentially more involved scaling relation among  $\nu_W$  and  $\eta$  exists still holds.

Finally, we determine the possible nontrivial fixed-point superpotentials. Solving the coupled fixed-point equations for the dimensionless superpotential as well as the anomalous dimension self-consistently, we find a discrete set of fixed-point superpotentials with an increasing number of nodes, see Fig. 2 (bottom). The number of RG relevant directions at the fixed points in addition to the  $a^2$  direction is equal to the number of nodes. The superpotential with no nodes is the next-to-leading order analogue of the maximally IR-stable fixed point discussed above. Our quantitative next-to-leading order estimates of the critical exponent  $\nu_W$  and the anomalous dimension  $\eta$  for the fixed-point superpotentials of Fig. 2 (bottom) are summarized in Tab. I. As these quantities are universal, these results should constitute regulator-independent predictions. Of course, the truncation of the effective action introduces an artificial regulator dependence, the size of which can be used for an error estimate

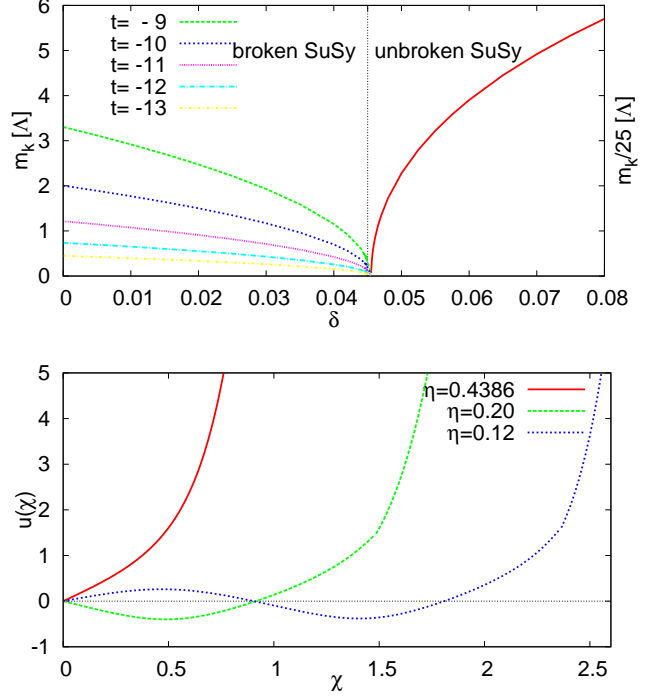


FIG. 2: *Top*: Scale dependence of the renormalized bosonic mass. *Bottom*: Fixed-point superpotentials at next-to-leading order derivative expansion,  $\chi = Z_k \phi$ ,  $u = w_k''/Z_k^2$ .

of our numerical results. From our regulator studies, we found a maximum variation of 10% on the positive critical exponents. Subleading negative exponents can show larger variations and thus are difficult to estimate reliably, whereas the leading exponents vary even less, if at all [19].

Each fixed point constitutes a universality class and defines a distinct asymptotically safe UV completion of the Wess-Zumino model, i.e., RG trajectories emanating from different fixed points correspond to different theories. Our findings are reminiscent of those in 2D scalar field theories [16, 17], where the fixed points of the effective potential can be related to conformal field theories [18]. The connection between the present supersymmetric models at their fixed points and conformal field theories remains an interesting question.

To summarize, we have constructed a manifestly supersymmetric flow equation for 2D  $\mathcal{N} = 1$  Wess-Zumino models. These models have nontrivial fixed-point superpotentials which can be classified by their relevant directions. All fixed points share a relevant direction in the form of a bosonic mass term. At next-to-leading order, the associated critical exponent  $\nu_W$  and the anomalous dimension  $\eta$  satisfy a superscaling relation which has no analogue in Ising-type spin systems.

The fixed point of this relevant direction corresponds to the critical point separating the supersymmetric from

the broken phase. The initial condition for this relevant direction defines a control parameter, such that supersymmetry breaking can be understood as a quantum phase transition. For the Gaussian Wess-Zumino model, we observe that the control parameter stays finite even at arbitrarily large coupling in accord with a general argument by Witten. In the broken phase, our superscaling relation predicts that the measured bosonic mass is proportional to the momentum scale set by the detector.

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